$$\frac{d\rho}{d\theta}=\pm\frac{a\rho}{\sqrt{(b^2-a^2)}}, \text{ or } \kappa\rho=e^{\pm[a/\sqrt{(b^2-a^2)}]\theta}.$$

The cylinder has therefore for its right section a logarithmic spiral or rather two arranged in opposite directions.

3. This curious result may be stated in the following way. If an elliptical hole be cut out of a sheet of paper and the sheet wrapped about a sphere the diameter of which equals the minor axis of the ellipse the sheet wraps up into a cylinder a cross section of which is a logarithmic spiral. A model illustrating this relation between the ellipse and the spiral is easily constructed.

BERKELEY, *April, 1904.*

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

200. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

No matter what value x be given, the *numerical* value of the expression $(x+2)/(2x^2+3x+6)$ can never exceed $\frac{1}{3}$.

II. Solution by G. W. GREENWOOD, M.A., Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill.

We know that $y+1/y \nless 2$.

$\therefore z/2+2/z \nless 2.\quad 2z+8/z \nless 8.\quad 2z+8/z-5 \nless 3.\quad \dfrac{2z^2-5z+8}{z} \nless 3.$

Now put $z=x+2$ and we get

$$\frac{2x^2+3x+6}{x+2} \nless 3;\ i.\ e.,\ \frac{x+2}{2x^2+3x+6} \ngtr \tfrac{1}{3}.$$

202. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Express in the form of radicals the roots of the equation

$$x^9+9mx^7+27m^2x^5+30m^3x^3+9mx+2r=0.$$

I. Solution by A. H. HOLMES, Brunswick, Maine.

Writing $x^3+3mx=y$ the equation becomes

$$y^3+3m^3y+2r=0 \text{.........} (1).$$

Let $s_1=[-r+\surd(m^9+r^2)]^{\frac{1}{3}}$, $s_2=[-r-\surd(m^9+r^2)]^{\frac{1}{3}}$, and the roots of (1) become $y_1=s_1+s_2$, $y_2=\frac{1}{2}(s_1+s_2)+\frac{1}{2}\surd(-3)(s_1-s_2)$, $y_3=-\frac{1}{2}(s_1+s_2)-\frac{1}{2}\surd(-3)(s_1-s_1)$.

$\therefore x^3+3mx=y_1$ or y_2 or y_3.........(2).

The nine roots of the three cubics (2) give the required solution. We let

$$s_3=\left(-\frac{y_1}{2}+\surd(m^3+\tfrac{1}{4}y_1^2)\right)^{\frac{1}{3}},\quad s_4=\left(-\frac{y_1}{2}-\surd(m^3+\tfrac{1}{4}y_1^2)\right)^{\frac{1}{3}}$$

and we denote the corresponding radicals when y_1 is replaced by y_2, by s_5, s_6; and when y_1 is replaced by y_3 by s_7, s_8. The nine roots of the given equation are:

$x_1=s_3+s_4$, $\qquad x_4=s_5+s_6$,

$x_2=-\frac{1}{2}(s_3+s_4)+\frac{1}{2}\surd(-3)(s_3-s_4)$, $\qquad x_5=-\frac{1}{2}(s_5+s_6)+\frac{1}{2}\surd(-3)(s_5-s_6)$,

$x_3=-\frac{1}{2}(s_3+s_4)-\frac{1}{2}\surd(-3)(s_1-s_4)$, $\qquad x_6=-\frac{1}{2}(s_5+s_6)-\frac{1}{2}\surd(-3)(s_5-s_6)$,

$x_7=s_7+s_8$,

$x_8=-\frac{1}{2}(s_7+s_8)+\frac{1}{2}\surd(-3)(s_7-s_8)$,

$x_9=-\frac{1}{2}(s_7+s_8)-\frac{1}{2}\surd(-3)(s_7-s_8)$.

Similarly solved by Elmer Schuyler, and J. Scheffer.

II. Solution by the PROPOSER.

Let $x=y+z$. Then $x^9+9mx^7+27m^2x^5+30m^3x^3+9m^4x+2r=0$, becomes

$y^9+z^9+(yz+m)\,[9(y^7+z^7)+(36yz+27m)(y^5+z^5)+(84y^2z^2+105myz+30m^2)(y^3+z^3)+(126y^3z^3+189my^2z^2+81m^2yz+9m^3)(y+z)+2r=0.$

$\therefore y^9+z^9=-2r$ and $yz=-m$.

$\therefore y^{18}+2ry^9=m^9$. $\quad\therefore y^9=-r\pm\surd(r^2+m^9)$.

Let $a^9=-r+\surd(r^2+m^9)$, $b^9=-r-\surd(r^2+m^9)$.

Let $w=\dfrac{-1+\surd(-3)}{2}=\beta^3$.

Then the nine roots are $a+b$, $\beta a+\beta^8 b$, $\beta^2 a+\beta^7 b$, $\beta^3 a+\beta^6 b$, $\beta^4 a+\beta^5 b$, $\beta^5 a+\beta^4 b$, $\beta^6 a+\beta^3 b$, $\beta^7 a+\beta^2 b$, $\beta^8 a+\beta b$.

Also solved by L. E. Newcomb, and F. D. Posey.

203. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

The sum of a certain pair of roots of $x^4+ax^3+\left(\dfrac{2b}{a}+\dfrac{a^2}{4}\right)x^2+bx+c=0$ is equal to the sum of the remaining pair.

I. Solution by J. SCHEFFER, Hagerstown, Md.

The problem reduces to the problem of finding the value of m in the equation $a^4+ax^3+mx^2+bx+c=0$, subject to the condition that the sum of two roots equals the sum of the two others. Denoting the roots by α, β, γ, δ, we have $\alpha+\beta=\gamma+\delta=-\frac{1}{2}a$. Since $\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=-b$, we have by factoring,

$$\alpha\beta(\gamma+\delta)+(\alpha+\beta)\gamma\delta=-b,\text{ or }(\alpha\beta+\gamma\delta)\tfrac{1}{2}a=b.$$